Capacitors Voltage Balancing Modeling in Three Phase Flying Capacitor Converters with Booster

Vahid Dargahi and Abbas Shoulaie

Department of Electrical Engineering, Iran University of Science and Technology, Tehran 16846, Iran

vdargahi@elec.iust.ac.ir, shoulaie@iust.ac.ir

Abstract—This paper provides a mathematical model for voltage natural balancing process in three phase capacitor-clamped multilevel converters. The analysis leads to state-space model of the converters. State-space representation of converter can be utilized to investigate the start-up and steady states of internal flying capacitors voltages. To provide verification, numerical solutions for three phase capacitor-clamped multilevel converter’s analytic model are presented.

Keywords Three Phase Flying Capacitor Multicell Converter, Self-balancing, Voltage Balance Booster.

I. INTRODUCTION

As a consequence of reaching higher power and lack of its suitable ranked switches, multilevel converters were introduced in 1975 and have been continuously developed in recent years due to the necessity of increase in power level of industrial applications especially high power applications such as high power AC motor drives, active power filters, reactive power compensation and FACTS devices. The main reason is the capability of these topologies to handle voltage/power in the range of kilovolts/megawatts as a result of serial connection of power switches in these converters [1].

The concept of multilevel arises from acquiring a staircase output voltage waveform as voltage levels from input dc voltages by means of converter appropriate configuration and its proper switching pattern [2]-[3].

In comparison with the conventional two-level converters, multilevel ones excel at producing an output voltage comprising several steps with considerable enhancements to power quality, harmonic content, and efficiency. Multilevel converters have other appreciable advantages such as lower switching losses, lower voltage ratings of used semiconductor switches, reduction of output dv/dt stress and filter inductance, etc. [4]-[6].

The term multilevel starts with the three-level converter introduced by Nabae et al. The Neutral Point Clamped (NPC) converter, presented in the early 80’s, is a standard topology in industry on its 3-level version. However, for a higher number of levels, this topology has some drawbacks such as: voltage imbalance issue of the dc-link capacitors and the excessive use of clamping diodes [7].

Cascade multicell (CM) converters use a series connection of H-bridge modules. Dc link voltage of each H-bridge module must be an isolated one. Modularity, ease of extension the number of output voltage levels via adding new modules, reliability, and fault tolerant feature are the most notable advantages of these topologies [8]-[9].

Flying capacitor (FC) based converters use ladder connection of units called as ‘cells’. Each cell is composed of one flying capacitor and two complimentary power switches. Redundant switching states in flying capacitor based converters can be implemented to stabilize the voltage across flying capacitors at their requisite values. The difference between voltages across two adjacent flying capacitors determines the step value of staircase-form output voltage [10]-[12].

The FCM converter, and its derivative, the SM converter, have many advantageous properties for medium voltage applications, particularly the transformer-less operation and the ability to naturally maintain the clamping capacitors voltages at their target operating levels. This substantial property is called natural balancing and allows the construction of such converters with a large number of voltage levels. Natural self-balancing of the flying capacitors voltages occurs without any feedback control. A necessary condition for this phenomenon is that average current of the clamping capacitors must be zero. As a result, each cell must be controlled with the same duty cycle and a regular phase shifted progression along the cells. Generally, an output RLC filter (balance booster circuit), tuned to the switching frequency or multiple of that, is suggested to be connected across the load in order to accelerate this self-balancing process in the transient states. The FCM converter uses a series connection of “cells” comprising a flying capacitor and its associated complimentary switch pair and produces a switched voltage that is the sum of the individual cell states [13].

Despite of mentioned appreciable advantages, multilevel converters possess some following main drawbacks: increased number of isolated dc voltage sources, clamping diodes, capacitors and of power semiconductor switches accompanied
II. INSTANTANEOUS MODELING OF THREE PHASE FLYING CAPACITOR MULTICELL CONVERTERS IN STATE-SPACE REPRESENTATION

A. Fundamental Concepts of Flying Capacitors Multicell Converters

Flying capacitor multicell converters (FCMCs) which have been proposed by T.A. Meynard are relatively new breed of multilevel converters in comparison with conventional neutral point clamped (NPC) and cascade H-bridge (CHB) ones. A typical single phase configuration of FCMC is depicted in Fig 1. As illustrated, \( R \) cells in a FCMC are overlapped to form a required converter’s leg. Each cell consists of one voltage source (a dc voltage source equal to \( E \) in \( R \)th cell and capacitors possessing specific voltages in remaining cells) and two power semiconductor switches which are in complementary state to each other to avoid short-circuiting of voltage sources. Phase Shifted Carrier Sinusoidal Pulse Width Modulation (PSC-SPWM) technique is the most common control scheme which is applied to switching strategy of FCMCs to guaranty both best harmonic performance and voltage balancing mechanism in clamping capacitors. It should be noted that in a \( R \)-cell FCMC each switch sustains just a fraction of DC link voltage, i.e. \( E/R \). This \( R \)-cell configuration leads to \( R+1 \) levels of voltage with peak to peak value of \( E \) at the converter output. FCMCs are in preference to the NPC and CHB ones as considering appreciable advantages such as: modularity, non-interdependency of cells as fault occurs and ease of reaching higher voltage levels just by introducing new cells [1]-[10].

Control strategy, switches states and output voltage of a 4-cell-5-level FCMC are illustrated in Fig. 2 and Table-1, respectively.

<table>
<thead>
<tr>
<th>Output Voltage</th>
<th>State of Switches (( S_s, S_c, S_w ))</th>
<th>Number of States</th>
</tr>
</thead>
<tbody>
<tr>
<td>+0.5E</td>
<td>{1,1,1,1}</td>
<td>1</td>
</tr>
<tr>
<td>+0.25E</td>
<td>{(1,1,0,1),(1,0,1,1),(0,1,1,1)}</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>{(1,1,0,0),(1,0,0,1),(0,1,0,1)}</td>
<td>6</td>
</tr>
<tr>
<td>-0.25E</td>
<td>{(1,0,0,0),(1,0,1,0),(0,1,0,1)}</td>
<td>4</td>
</tr>
<tr>
<td>-0.5E</td>
<td>{0,0,0,0}</td>
<td>1</td>
</tr>
</tbody>
</table>

The output voltage of a \( R \)-cell FCM converter has \( R+1 \) levels and its frequency spectrum has the harmonics around \((R \times k \times f_{SW})^{th}\) harmonic where \( k \) and \( f_{SW} \) are the integer number and the switching frequency, respectively [11]-[14]. A typical three phase clamping capacitor multicell converter is shown in Fig 3.
The output voltage and current of the cells. Generally, an output voltage value of $E$ in each phase.

**B. Instantaneous Model of the Three Phase Converter in the State-Space Representation**

By utilizing proper switching pattern in FCMCs, their capacitor voltages would reach to the specific values which allow to constructing the desired output voltage levels. This property is known as voltage natural balancing mechanism and is achieved using PSC-SPWM switching technique in these converters [11]-[21].

Natural self-balancing process of the clamping capacitors voltages, as one of the advantages of FCMCs occurs without any feedback control. A necessary condition for this phenomenon is that average current of the flying capacitors must be zero. As a result, each cell must be controlled with the same duty cycle and a regular phase shift progression along the cells. Generally, an output RLC filter (balance booster circuit), tuned to the switching frequency or multiple of that, is suggested to be connected across the load in order to accelerate this self-balancing process in the start-up states. In this case, the dynamic of the self-balancing process depends on the impedance of balance booster circuit at the switching frequency. If the impedance at the switching frequency is high then the natural balancing is very slow and vice versa. The output RLC filter should be tuned to the switching frequency as follow [1]:

$$\sqrt{L_b \cdot C_b} = \frac{1}{2 \pi \cdot f_{sw}}$$  \hspace{1cm} (1)

Where, $f_{sw}$ is the switching frequency, $L_b$ and $C_b$ are inductance and capacitance of the output RLC booster circuit, respectively.

In this section a mathematical model for three phase capacitor-clamped converter will be presented to verify natural balancing property. In the derived model, capacitors voltages are state variables.

Utilization a numerical solution for proposed model differential equations leads to acquire transient and steady state response of flying capacitors voltages. According to Fig. 3, switching function of cell $\rho$ in each phase of the converter is defined as follows [15]-[21]:

$$H_{\rho, x}(t) = \begin{cases} +1 & \text{if } S_{\rho, x} \text{ is on} \\ -1 & \text{if } S_{\rho, x} \text{ is off} \end{cases} \quad \rho = 1, 2, 3, x \in \{a, b, c\}$$  \hspace{1cm} (2)

And its Double Fourier Series expansion can be expressed as follows [15]:

$$H_{\rho, x}(t) = M \cos(\delta)$$

$$\delta = \omega_f t + \varphi + (\rho - 1)2\pi R \frac{1}{\omega_s}, \varphi = \phi_I + \varphi_r + \varphi_c$$  \hspace{1cm} (3)

where $M$, $\omega_f$, $\phi_I$, $J_\rho(t)$, $\omega_c$, $\phi_c$ are modulation index, angular frequency of reference sinusoidal waveform, phase angle of reference sinusoidal waveform, $n^{th}$ order Bessel function of first kind, triangular carrier wave frequency angular frequency and triangular carrier waveform phase angle, respectively.

According to Fig. 3, output voltage and current of the converter based on mentioned switching functions can be obtained as follows:

$$V_{ao}(t) = V_{ao}(t) + V_{ao}(t)$$

$$V_{bo}(t) = V_{bo}(t) + V_{bo}(t)$$

$$V_{co}(t) = V_{co}(t) + V_{co}(t)$$

$$V_{av}(t) + V_{bv}(t) + V_{cv}(t) =$$

$$V_{av}(t) + V_{bv}(t) + V_{cv}(t) + 3V_{ao}(t)$$

$$V_{av}(t) = \frac{1}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} V_{ao}(t)$$

$$V_{av}(t) = \frac{1}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} V_{bo}(t)$$

$$V_{av}(t) = \frac{1}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} V_{cv}(t)$$
\[
\begin{align*}
\frac{dE_{p,a}(t)}{dt} &= \frac{1}{2Z_L}(H_{p+1,a}(t)-H_{p,a}(t))i_{a}(t) \\
\frac{dE_{p,b}(t)}{dt} &= \frac{1}{2Z_L}(H_{p+1,b}(t)-H_{p,a}(t))i_{b}(t) \\
\frac{dE_{p,c}(t)}{dt} &= \frac{1}{2Z_L}(H_{p+1,c}(t)-H_{p,a}(t))i_{c}(t)
\end{align*}
\]

Where, \( Z_L \) is a series connection of resistance \( (r) \) and inductance \( (l) \) and \( C_p \) is capacitance of flying capacitors.

Three phase clamping capacitor multicell converter’s state-space representation can be written as follows:

\[
\begin{align*}
\frac{dE_{p,a}(t)}{dt} &= \frac{1}{2Z_L}(H_{p+1,a}(t)-H_{p,a}(t)) \times \left( \sum_{j=1}^{\frac{L}{3}} \left( \lambda_{j,a}(t) - \lambda_{j+1,a}(t) \right) E_{j,a}(t) \right) \\
\frac{dE_{p,b}(t)}{dt} &= \frac{1}{2Z_L}(H_{p+1,b}(t)-H_{p,a}(t)) \times \left( \sum_{j=1}^{\frac{L}{3}} \left( \lambda_{j,b}(t) - \lambda_{j+1,b}(t) \right) E_{j,b}(t) \right) \\
\frac{dE_{p,c}(t)}{dt} &= \frac{1}{2Z_L}(H_{p+1,c}(t)-H_{p,a}(t)) \times \left( \sum_{j=1}^{\frac{L}{3}} \left( \lambda_{j,c}(t) - \lambda_{j+1,c}(t) \right) E_{j,c}(t) \right)
\end{align*}
\]

\[
E_{C_i}(t) = \begin{cases} 
B_{i,a}(t) & i, j = 1: R-1 \\
B_{i,b}(t) & x, y \in \{a, b, c\} \\
B_{i,c}(t) & x, y \in \{a, b, c\}
\end{cases}
\]

\[
\lambda_{j,a}(t) = \frac{1}{6C_{i,a}} \left( H_{j+1,a}(t) - H_{j,a}(t) \right) \left( \lambda_{j,a}(t) - \lambda_{j+1,a}(t) \right) + \frac{1}{6C_{i,a}} \left( H_{j+1,a}(t) - H_{j,a}(t) \right) \times E
\]

\[
\lambda_{j,b}(t) = \frac{1}{12} \left( H_{j+1,b}(t) - H_{j,a}(t) \right) \times E
\]

\[
\lambda_{j,c}(t) = \frac{1}{12} \left( H_{j+1,c}(t) - H_{j,a}(t) \right) \times E
\]

\[
\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{4}{m \pi r \xi_{m,n}} \sin \left( \left( m + \frac{\pi}{2} \right) \frac{m \pi M}{2} \right) \cos \left( m \vartheta + n \vartheta - \tan^{-1} \beta_{m,n} \right)
\]

\[
\xi_{m,n} = \sqrt{\left( 1 + \beta_{m,n}^2 \right)}
\]

\[
\beta_{m,n} = \frac{\left( m \omega + n \omega \right) l}{r}
\]

To consider the balance booster circuit effect on the dynamic of the self-balancing process following modification should be applied to the acquired state-space equations:
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\[
A_{x,y}(t) = \begin{cases} 
\frac{1}{6C_{x,y}}(H_{x+1,y}(t) - H_{x,y}(t)) & x \neq y; \\
\frac{1}{12C_{x,y}}(H_{x+1,y}(t) - H_{x,y}(t)) & x = y;
\end{cases}
\]

\[
\lambda_{x,y}^{n,t}(t) = \frac{H_{x,y}^{n,t}(t)}{Z_b} = \frac{M}{R_b\Omega_{m,n}} \cos(\delta - \tan^{-1}\Psi_{m,n})
\]

\[
+ \sum_{m=-1}^{\infty} \sum_{n=-\infty}^{\infty} \frac{4}{m\pi R_b C_{m,n}} \sin\left(\frac{m\pi}{2}\right) J_n\left(\frac{m\pi}{2}M\right) 
\cos(m\vartheta + n\delta - \tan^{-1}\Psi_{m,n})
\]

\[
\Omega_{m,n} = \sqrt{1 + \Psi_{m,n}^2}
\]

\[
\Psi_{m,n} = \left(\frac{m\omega + n\omega_l}{C_b} \right) \frac{(m\omega + n\omega_l) C_b}{R_b}
\]

### III. NUMERICAL SOLUTION RESULTS

To provide verification to the elaborated state-space representation of the three phase FCMCs, numerical solution is utilized to solve differential equations of 2-cell-3-level and 3-cell-4-level three phase converters. Transient and steady state of internal flying capacitors voltages of mentioned converters acquired from state-space numerical solution are shown in Figs. 4-9. System parameters used in numerical solution are given in Tables 2-4.

Table-2: Parameters used in numerical solution of state-space representation of 2-cell-3-level three phase FCMC.

<table>
<thead>
<tr>
<th>System Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC voltage (E)</td>
<td>600 V</td>
</tr>
<tr>
<td>Internal flying capacitors (C)</td>
<td>560 uF</td>
</tr>
<tr>
<td>PSCSPWM carrier frequency (fsw)</td>
<td>5 kHz</td>
</tr>
<tr>
<td>Fundamental output voltage frequency</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Modulation index</td>
<td>0.8</td>
</tr>
<tr>
<td>Resistive-inductive load (r-l)</td>
<td>10 Ω – 80 mH</td>
</tr>
<tr>
<td>Booster circuit(RLC)</td>
<td>200 Ω – 6.97 uH-144 uF</td>
</tr>
</tbody>
</table>

Table-3: Parameters used in numerical solution of state-space representation of 2-cell-3-level three phase FCMC.

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</tr>
</tbody>
</table>

Fig. 4. Transient and steady state of internal flying capacitors voltages of a 2-cell-3-level three phase FCMC acquired from state-space numerical solution for resistive-inductive load in parallel with RLC booster circuit.

Fig. 5. Start-up state of internal flying capacitors voltages of a 2-cell-3-level three phase FCMC acquired from state-space numerical solution for resistive-inductive load in parallel with RLC booster circuit.
Table 4: Parameters used in numerical solution of state-space representation of 3-cell-4-level three phase FCMC.

<table>
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</tr>
<tr>
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</tr>
<tr>
<td>Fundamental output voltage frequency</td>
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</tr>
<tr>
<td>Modulation index</td>
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</tr>
<tr>
<td>Resistive-inductive load (r-l)</td>
<td>10 Ω – 50 mH</td>
</tr>
<tr>
<td>Booster circuit (RLC)</td>
<td>20 Ω – 6.97 μH – 144 μH</td>
</tr>
</tbody>
</table>

Numerical solutions predict the start-up and steady state of clamping capacitors voltages precisely and admit the proposed model for three phase clamping capacitor multicell converters. It is worth noting that the value of resistor in the balance booster circuit plays an important role in start-up state of flying capacitors voltages and its reduction accelerates the self-balancing process and vice versa.

IV. CONCLUSION

Clamping capacitor multicell converters are very interesting for high-power/medium-voltage applications, for considerably improvement of the output voltage frequency spectrum, voltage natural balancing of clamping capacitors and fault tolerance.

This paper presents a mathematical model for three phase clamping capacitor multicell converters. In the proposed model the effect of balance booster circuit which is usually connected in parallel with load to accelerate the self-balancing process of flying capacitors, is considered which has not been reported in
literature. Numerical solutions confirm the validity of proposed model for three phase flying capacitor multilevel converters.

REFERENCES


